A Surrogate-assisted Partial Optimization for Expensive Constrained Optimization Problems

Kei NISHIHARA, Masaya NAKATA   Yokohama National University, Japan

YNU YOKOHAMA National University

Mail   Paper (PDF)

Background

- **Constrained Optimization Problem (COP)**
  - Definition
    Single-objective minimization problems with inequality constraints are focused.
    $$\text{Minimize } f(\boldsymbol{x}) \quad \text{s.t. } g_m(\boldsymbol{x}) \le 0, m = 1,2,\ldots,M$$
  - Feasibility
    A solution $\boldsymbol{x}$ is feasible if the degree of constraint violation $G(\boldsymbol{x})$ meets
    $$G(\boldsymbol{x}) = \sum_{m=1}^{M} \max(g_m(\boldsymbol{x}), 0) = 0.$$
- **Surrogate-assisted Evolutionary Algorithm (SAEA)**
  - Expensive COPs (ECOPs) are widely seen in the real world.
    Function evaluations (FEs) are computationally and/or financially expensive.
  - SAEAs are a representative methodology for ECOPs.
    Machine learning models act as surrogates for parts of expensive FEs.
    Thus, SAEAs can save the number of FEs.
  - Most SAEAs construct a response surface set (RSS).
    An RSS is a set of approximation models of the objective and constraint functions.
    $$RSS = \{\hat{f}(\boldsymbol{x}), \hat{g}_1(\boldsymbol{x}), \hat{g}_2(\boldsymbol{x}), \ldots, \hat{g}_M(\boldsymbol{x})\}$$

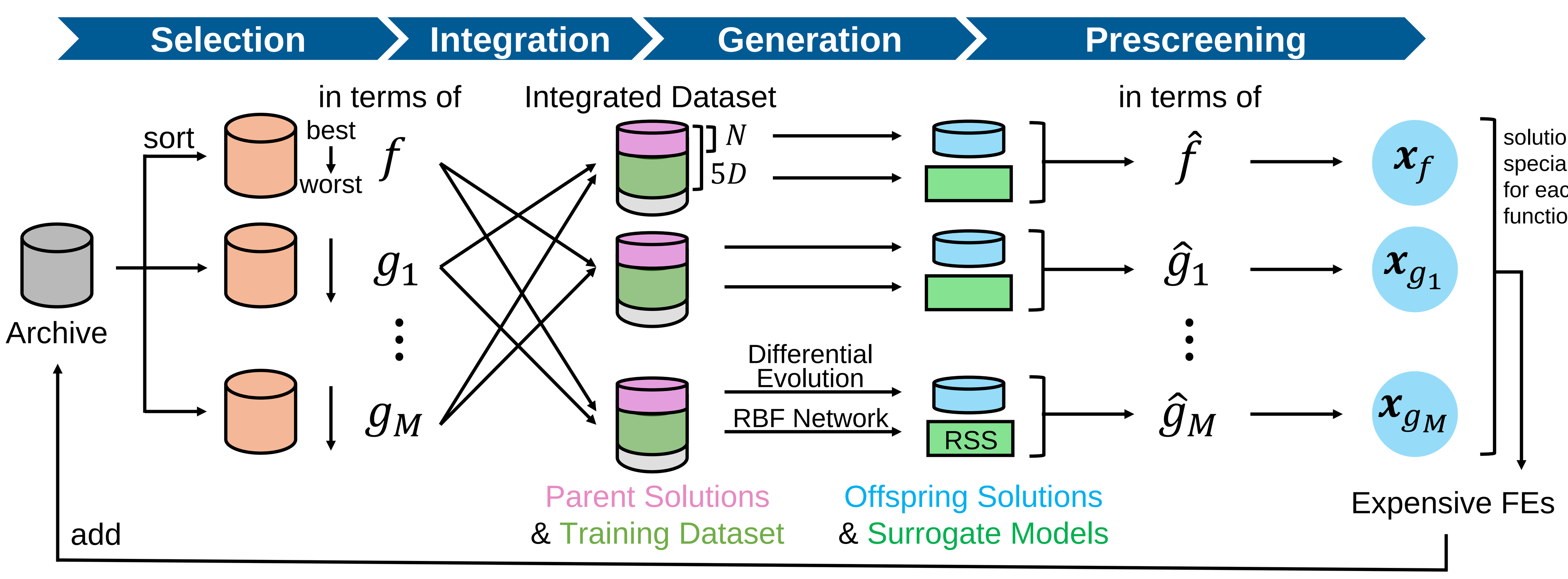
Related Work

- **How to use an RSS in the existing SAEAs**
  $\hat{G}(\boldsymbol{x}) = \sum_{m=1}^{M} \max(\hat{g}_m(\boldsymbol{x}), 0)$ approximation of $G(\boldsymbol{x})$
  - Feasibility Rule [Deb 00]
    $$\boldsymbol{x}^* = \begin{cases} \arg\min_{\boldsymbol{x}\in\mathcal{F}} \hat{f}(\boldsymbol{x}), & \mathcal{F} = \{\boldsymbol{x} \mid \hat{G}(\boldsymbol{x}) = 0\} \neq \emptyset \text{ (solutions expected to be feasible)} \\ \arg\min \hat{G}(\boldsymbol{x}), & \text{otherwise} \end{cases}$$
    examples: SA-DECVFR [Miranda-Varela+ 18], GLoSADE [Wang+ 19], SACCDE [Yang+ 20], FMSADE [Chu+ 20], and SA-TSDE [Liu+ 23]
  - Penalty Function [Homaifar+ 94]
    $$\hat{F}(x) = \hat{f}(\boldsymbol{x}) + \lambda\hat{G}(\boldsymbol{x})$$
    example: MPMLS [Li+ 21]
- **Solutions are prescreened only with $\hat{G}(\boldsymbol{x})$,**
  i.e., aggregation of $\hat{g}_m$s, although each $\hat{g}_m$ can be independently utilized.
  - ✓ **The feasibility of solutions is easily estimated.**
  - ✕ **Errors of $\hat{g}_m - g_m$ accumulate in $\hat{G}$.**
  - ✕ **The differences in scales between $\hat{g}_m$s are ignored.**
    Small scale $\hat{g}_m$s: Improvement is prevented by larger scale $\hat{g}_m$s.
    Large scale $\hat{g}_m$s: Constraint handling effects scatter to other trivial $\hat{g}_m$s.
  - ✕ **The g_ms are not always correlated with each other.**

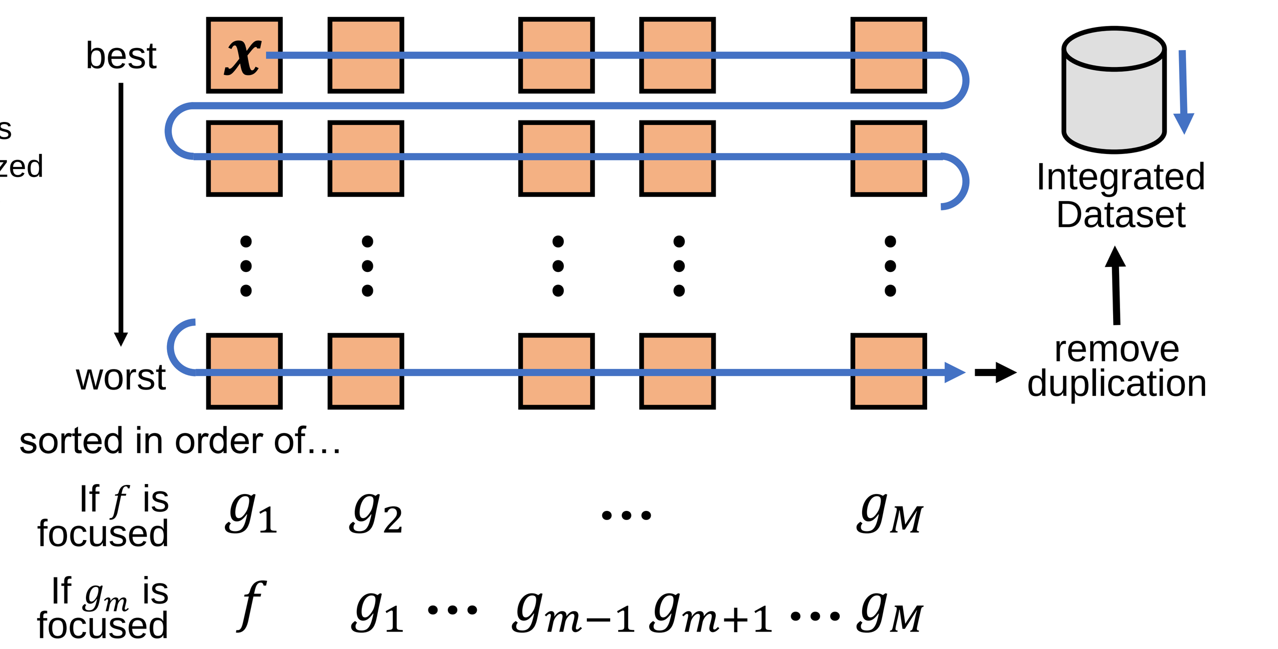
Proposed Algorithm: Surrogate-assisted Partial Optimization (SAPO)

➢ Inspired by Partial Differential Equation $\frac{\partial y}{\partial x}$
Focusing on one element improves the efficiency of structure analysis or optimization [Liu+ 21][Evans 22].

- **An SAEA that partially (independently) optimizes each objective/constraint in turn**
  - **Framework**



  - **Advantages**
    - ✓ **Each f or g_m is directly improved thus a greater number of FEs is saved.**
    - ✓ **Multiple criteria to prescreen offspring keep solution diversity high.**
    - ✓ **SAPO can handle ECOPs where the scale of the constraints vary widely or the constraints are not correlated with each other.**
  - **Detailed Procedures**
    - **Integration**



    - **Prescreening**
      If f is focused
      Apply the feasibility rule to offspring solutions
      If g_m is focused
      $$\boldsymbol{x}^* = \begin{cases} \arg\min_{\boldsymbol{x}\in\mathcal{G}} \hat{f}(\boldsymbol{x}), & \mathcal{G} = \{\boldsymbol{x} \mid \hat{g}_m(\boldsymbol{x}) \le 0\} \neq \emptyset \text{ (solutions that satisfy } \hat{g}_m) \\ \arg\min \hat{g}_m(\boldsymbol{x}), & \text{otherwise} \end{cases}$$

Experiment

Experimental Design: IEEE CEC2017 single-objective constrained real-parameter benchmark suite ($D \in [30, 50, 100\}$) [Wu+ 17], Maximum number of FEs = 3,000, Number of runs = 31

Parameter Settings of SAPO: $N = 100, F = 0.5, CR = 0.9, N_{init} = 100\ (D \in \{30, 50\}), 200\ (D = 100), kernel = cubic.$

- **Comparison with state-of-the-art SAEAs**

best | worst | no feasible solution is found in all algorithms

(n): number of successful runs among 31 runs

Wilcoxon's rank-sum test (significance level = 0.05)
+ : Compared algorithm outperforms
- : Compared algorithm underperforms
~ : Cannot find significance

Fitness values

$D = 30$

Problem (Number of Constraints)	GLoSADE	FMSADE	MPMLS	SA-TSDE	SAPO
F1 (1)	9.557e+03 −	*4.164e+04* −	2.499e+04 −	7.389e+03 −	**5.400e+03**
F2 (1)	4.032e+03 −	*(1)* −	5.468e+03 −	3.070e+03 −	**2.110e+03**
F4 (2)	3.883e+02 −	*4.060e+02* −	1.973e+02 ∼	2.148e+02 −	**1.888e+02**
F5 (2)	4.178e+01 −	*3.183e+02* −	**2.895e+01** ∼	5.707e+01 −	3.251e+01
F12 (2)	1.517e+02 −	*(0)* −	1.559e+01 −	(30) −	**1.261e+01**
F13 (3)	*(0)* −	*(0)* −	*(0)* −	**(7)** ∼	**(7)**
F20 (2)	9.801e+00 ∼	*1.002e+01* ∼	**9.589e+00** +	9.920e+00 ∼	9.877e+00
F21 (2)	*(0)* −	*(0)* −	(29) −	*(0)* −	**1.075e+01**
F22 (3)	(0) ∼	(0) ∼	(0) ∼	(0) ∼	(0)
+/−/∼	0/**7**/2	0/**7**/2	1/**5**/3	0/**6**/3	-
Ave. Rank	3.222	*4.556*	2.556	3.056	**1.611**

$D = 50$

Problem (Number of Constraints)	GLoSADE	FMSADE	MPMLS	SA-TSDE	SAPO
F1 (1)	3.038e+04 −	*1.064e+05* −	6.341e+04 −	3.852e+04 −	**2.547e+04**
F2 (1)	1.707e+04 −	*(0)* −	2.087e+04 −	1.487e+04 −	**1.056e+04**
F4 (2)	*6.819e+02* −	6.686e+02 −	4.574e+02 −	3.507e+02 −	**3.100e+02**
F5 (2)	2.444e+03 −	*(27)* −	1.210e+02 −	1.879e+03 −	**9.113e+01**
F12 (2)	*(0)* −	*(0)* −	1.052e+02 −	*(0)* −	**1.429e+01**
F13 (3)	(0) ∼	(0) ∼	(0) ∼	(0) ∼	(0)
F20 (2)	1.824e+01 ∼	*1.882e+01* ∼	**1.803e+01** ∼	1.860e+01 ∼	1.838e+01
F21 (2)	*(0)* −	*(0)* −	*(0)* −	*(0)* −	**(30)**
F22 (3)	(0) ∼	(0) ∼	(0) ∼	(0) ∼	(0)
+/−/∼	0/**6**/3	0/**6**/3	0/**6**/3	0/**6**/3	-
Ave. Rank	3.278	*4.167*	2.833	3.056	**1.667**

$D = 100$

Problem (Number of Constraints)	GLoSADE	FMSADE	MPMLS	SA-TSDE	SAPO
F1 (1)	1.644e+05 ∼	*4.964e+05* −	2.352e+05 −	**1.597e+05** ∼	1.624e+05
F2 (1)	1.314e+05 −	*(0)* −	9.185e+04 −	9.066e+04 −	**7.763e+04**
F4 (2)	1.491e+03 −	*1.523e+03* −	1.257e+03 −	8.659e+02 −	**7.750e+02**
F5 (2)	4.271e+04 −	*(25)* −	2.094e+03 −	2.385e+04 −	**1.384e+03**
F12 (2)	*(0)* −	*(0)* −	*(0)* −	*(0)* −	**(19)**
F13 (3)	(0) ∼	(0) ∼	(0) ∼	(0) ∼	(0)
F20 (2)	4.007e+01 +	4.067e+01 ∼	**3.969e+01** +	*4.113e+01* ∼	4.093e+01
F21 (2)	(0) ∼	(0) ∼	(0) ∼	(0) ∼	(0)
F22 (3)	(0) ∼	(0) ∼	(0) ∼	(0) ∼	(0)
+/−/∼	1/**4**/4	0/**5**/4	1/**5**/3	0/**4**/5	-
Ave. Rank	3.278	*3.944*	2.833	2.833	**2.111**

Wilcoxon's rank-sum test (+/-/~)

D	FE	GLoSADE	FMSADE	MPMLS	SA-TSDE
30	300	0/**3**/6	0/**4**/5	0/**3**/6	**1**/**1**/7
	500	0/**5**/4	0/**5**/4	0/**3**/6	**2**/1/6
	1,000	0/**6**/3	0/**6**/3	0/**5**/4	1/**3**/5
	2,000	0/**7**/2	0/**7**/2	1/**7**/1	0/**5**/4
	3,000	0/**7**/2	0/**7**/2	1/**5**/3	0/**6**/3
50	300	0/**3**/6	1/**3**/5	0/**3**/6	**2**/**2**/5
	500	0/**3**/6	0/**4**/5	0/**3**/6	**2**/1/6
	1,000	0/**5**/4	0/**5**/4	0/**5**/4	**2**/**2**/5
	2,000	0/**6**/3	0/**7**/2	0/**6**/3	0/**5**/4
	3,000	0/**6**/3	0/**6**/3	0/**6**/3	0/**6**/3
100	300	0/**2**/7	0/**3**/6	0/**1**/8	**2**/**2**/5
	500	1/**2**/6	0/**3**/6	1/**2**/6	**2**/**2**/5
	1,000	1/**3**/5	0/**4**/5	1/**2**/6	**2**/**2**/5
	2,000	0/**3**/6	0/**4**/5	1/**3**/5	1/**2**/6
	3,000	1/**4**/4	0/**5**/4	1/**5**/3	0/**4**/5

✓ **SAPO found more and better feasible solutions within a smaller number of FEs than compared SAEAs thanks to the proposed partial optimization.**

- **Ablation Studies**
  - **Three variants of SAPO were compared with the original SAPO.**
    - **VUA:** Only the approximation of G, i.e., $\hat{G}$ is used. – A similar setting to the existing SAEAs.
    - **VTO:** Only the objective function $\hat{f}$ is focused. – To confirm the need to focus on $\hat{g}_m$s.
    - **VTC:** Only the constraint functions $\hat{g}_m$s are focused. – To confirm the need to focus on $\hat{f}$.
  - **Results – The original SAPO outperformed the variants towards the end of optimization.**
    - **VUA & VTO :** These variants suffered from the premature convergence.
      – Many FEs were used to get already found feasible solutions and infeasible solutions with good $\hat{f}$, respectively.
    - **VTC :** The fitness values f did not improve. – Focusing only $\hat{g}_m$s was not suitable.

(significance level = 0.05)
+ : Variant outperforms
- : Variant underperforms
~ : Cannot find significance

Wilcoxon's rank-sum test (+/-/~)

FE	$D=30$ VUA	$D=30$ VTO	$D=30$ VTC	$D=50$ VUA	$D=50$ VTO	$D=50$ VTC	$D=100$ VUA	$D=100$ VTO	$D=100$ VTC
300	**3**/1/5	**3**/0/6	1/**4**/4	**2**/0/7	**3**/0/6	0/**3**/6	**2**/0/7	**4**/0/5	0/**4**/5
500	1/**3**/5	**3**/0/6	0/**8**/1	**5**/0/4	**5**/0/4	1/**4**/4	**4**/0/5	**4**/0/5	0/**4**/5
1,000	0/**5**/4	0/**5**/4	0/**8**/1	**4**/0/5	**4**/0/5	0/**6**/3	**6**/0/3	**4**/0/5	0/**6**/3
2,000	0/**5**/4	0/**4**/5	0/**8**/1	**2**/1/6	**2**/**2**/5	0/**7**/2	**2**/1/6	**2**/1/6	0/**7**/2
3,000	0/**5**/4	0/**3**/6	0/**8**/1	0/**3**/6	0/**3**/6	0/**6**/3	1/**3**/5	1/**2**/6	0/**6**/3

Future Work
- Adaptive selection of $\hat{f}$ and $\hat{g}_m$s to be optimized
- Extension of SAPO for multi-objective ECOPs