Complex & Intelligent Systems

Emulation-based Adaptive Differential Evolution: Fast and Auto-tunable Approach for Moderately Expensive Optimization Problems

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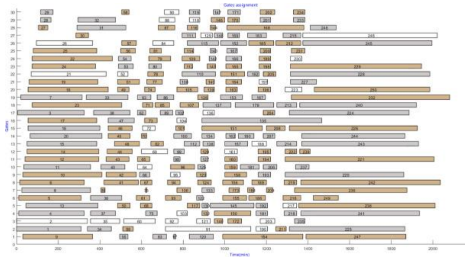
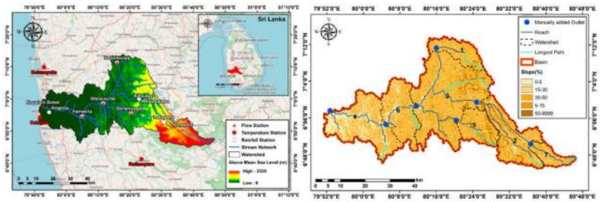
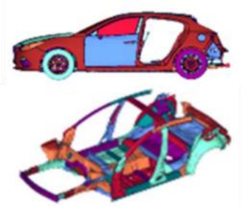
Background

Background

- **Expensive Optimization Problem (EOP) in real-world**
  - Function Evaluation (FE) is computationally or financially expensive in EOPs. [Shan+ 10]
  - The number of FE is restricted due to limited budget.

- **Classification of EOP**

	Non-expensive (Normal)	**Moderately EOP (M-EOP)**	**EOP**
Problem Example	 Airport Gate Allocation [Deng+ 22]	 Automatic Calibration of Watershed Models [Makumbura+ 22]	 Vehicle Structure Optimization [Oyama+ 17]
Evaluation Time Ex.	Less than 1 second	2 minutes	20 hours
Max. Number of FEs	Hundreds of thousands	Several thousand	Hundreds to a few thousand
Main approach	Evolutionary Algorithm (EA)	Not adequately researched	Surrogate-assisted EA (SAEA) (to be explained in detail next)

Research Purpose

SAEA: Main Approach for EOPs

- Usefulness in EOPs (Hundreds to a few thousand FEs)
  - Surrogates of the objective function are constructed using machine learning (ML).
  - Surrogates identify expected-to-improve solutions without FE.

    e.g., Expected Improvement (EI) metric [Jones+ 98]

$$\mathrm{E}[I(\mathbf{x})] = (f_{\min} - \hat{y})\Phi\left(\frac{f_{\min} - \hat{y}}{s}\right) + s\phi\left(\frac{f_{\min} - \hat{y}}{s}\right)$$

- Limitations in M-EOPs (Several thousand FEs)
  1. Premature convergence [Sun+ 15]

     SAEAs have strong exploitation nature.
  2. Time-consuming [Briffoteaux 22]

     ML models are repeatedly construct/used.
     Reducing the runtime is crucial in M-EOPs.
  3. Fixed parameter configuration [Lobo+ 07]

     Advance fine-tuning is hindered in (M-)EOPs although tuning configuration is important.

Need for an approach for M-EOPs

Research Purpose	Proposing a 1) High-performance, 2) Fast, and 3) Auto-tunable EA for M-EOPs.

Research Approach

- **Auto-tunable and Computationally Efficient Adaptive EA**
  - Adaptive EA
    - Auto-tunable: Parameter configurations are automatically controlled during a run.
    - Much faster than SAEAs: Adaptive EAs do not use ML techniques.
    - Slow convergence: Most are not for (M-)EOPs, i.e., hundreds of thousands of FEs.
  - Idea to boost convergence speed

Existing Adaptive EAs		Proposed Algorithm
Trial-and-error Adaptation Configurations are updated based on ones generated good solutions in past generations.	→	**Adaptation with Prior Validation** Pre-screen candidate configurations before use without FE.
Individual-based Adaptation The effectiveness of each configuration is usually validated with only one sample.	→	**Subpopulation-based Adaptation** The effectiveness of configurations are carefully validated using multiple samples.

Preliminary

- **Differential Evolution (DE)**

[Storn+ 97]

  - A population-based evolutionary algorithm

Initialize $\mathcal{P} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_N\}$

while *termination criteria* are not met **do**

$t = t + 1$

for $i = 1$ **to** N **do**

$\boldsymbol{v}_i \leftarrow \mathit{Mutation}(\mathcal{P}, \theta_{v,i}, \theta_{F,i})$

$\boldsymbol{u}_i \leftarrow \mathit{Crossover}(\boldsymbol{x}_i, \boldsymbol{v}_i, \theta_{u,i}, \theta_{CR,i})$

for $i = 1$ **to** N **do**

$$\boldsymbol{x}_i \leftarrow \begin{cases} \boldsymbol{u}_i & \text{if } f(\boldsymbol{u}_i) \leq f(\boldsymbol{x}_i) \\ \boldsymbol{x}_i & \text{otherwise} \end{cases}$$

Mutation: generate a mutant solution $\boldsymbol{v}_i$ for each $\boldsymbol{x}_i$

parameter Scaling factor $\theta_F \in [0, 1]$

strategy (mutation strategy)

rand/1	$\boldsymbol{v}_i = \boldsymbol{x}_{r1} + \theta_F(\boldsymbol{x}_{r2} - \boldsymbol{x}_{r3})$
rand/2	$\boldsymbol{v}_i = \boldsymbol{x}_{r1} + \theta_F(\boldsymbol{x}_{r2} - \boldsymbol{x}_{r3}) + \theta_F(\boldsymbol{x}_{r4} - \boldsymbol{x}_{r5})$
best/1	$\boldsymbol{v}_i = \boldsymbol{x}_{best} + \theta_F(\boldsymbol{x}_{r1} - \boldsymbol{x}_{r2})$
best/2	$\boldsymbol{v}_i = \boldsymbol{x}_{best} + \theta_F(\boldsymbol{x}_{r1} - \boldsymbol{x}_{r2}) + \theta_F(\boldsymbol{x}_{r3} - \boldsymbol{x}_{r4})$
current-to-rand/1	$\boldsymbol{v}_i = \boldsymbol{x}_i + \theta_F(\boldsymbol{x}_{r1} - \boldsymbol{x}_i) + \theta_F(\boldsymbol{x}_{r2} - \boldsymbol{x}_{r3})$
current-to-best/1	$\boldsymbol{v}_i = \boldsymbol{x}_i + \theta_F(\boldsymbol{x}_{best} - \boldsymbol{x}_i) + \theta_F(\boldsymbol{x}_{r1} - \boldsymbol{x}_{r2})$
current-to-pbest/1	$\boldsymbol{v}_i = \boldsymbol{x}_i + \theta_F(\boldsymbol{x}_{pbest} - \boldsymbol{x}_i) + \theta_F(\boldsymbol{x}_{r1} - \widetilde{\boldsymbol{x}_{r2}})$
rand-to-best/1	$\boldsymbol{v}_i = \boldsymbol{x}_{r1} + \theta_F(\boldsymbol{x}_{best} - \boldsymbol{x}_{r1}) + \theta_F(\boldsymbol{x}_{r2} - \boldsymbol{x}_{r3})$

Crossover: generate a trial solution $\boldsymbol{u}_i$ from $\boldsymbol{x}_i$ and $\boldsymbol{v}_i$

parameter Crossover rate $\theta_{CR} \in [0, 1]$

strategy (crossover strategy)

binomial : $u_{i,j} = \begin{cases} v_{i,j}, & \text{if } (rand(0,1) \leq \theta_{CR}) \text{ or } (j = j_{rand}) \\ x_{i,j}, & \text{otherwise} \end{cases}$

exponential : a method like one/two-point crossover in GA

Selection: select next $\boldsymbol{x}_i$ from current $\boldsymbol{x}_i$ and $\boldsymbol{u}_i$

Related Works

- **Adaptive/Surrogate-assisted DE**

  Indiv.: individual-based adaptation

  Each solution $\boldsymbol{x}_i$ has its own configuration $\boldsymbol{\theta}_i$.

  Subpop.: subpopulations-based adaptation

  Solutions in a subpopulation use same $\boldsymbol{\theta}$.

  - Many adaptive DEs are Indiv.
    - Recently, Subpop. begins to gain popularity.
  - Some surrogate-assisted DEs incorporate adaptive mechanism into SAEAs.
    - However, they are usually Indiv.

- **Position of Proposed Algorithm**

  Subpop. and for M-EOPs

Algorithm	Adaptation Style	D (Prob. Dim.)	$FE_{\max}$ (Max. # of FEs)
adaptive DEs			
jDE [3]	Indiv.	{2, 4, 30}	10,000–20,000,000
FDSADE [53]	Indiv.	{2, 4, 30}	50,000
ISADE [15]	Indiv.	30	300,000
JADE [63]	Indiv.	{2, 3, 4, 6, 30, 100}	6,000–8,000,000
MDE_pBX [14]	Indiv.	{30, 50, 100}	$D\times$10,000
SHADE [50]	Indiv.	30	300,000
L-SHADE [52]	Indiv.	{10, 30, 50, 100}	$D\times$10,000
jSO [4]	Indiv.	{10, 30, 50, 100}	$D\times$10,000
SaDE [42]	Indiv.	{10, 30}	100,000–500,000
CoDE [57]	Indiv.	30	300,000
EPSDE [35]	Indiv.	{10, 30, 50}	$D\times$10,000
CSDE [48]	Indiv.	{30, 50, 100}	$D\times$10,000
AL-SHADE [24]	Indiv.	{10, 30, 50}	$D\times$10,000
DE-DDQN [45]	Indiv.	{10, 30}	10,000
FLDE [49]	Indiv.	{10, 30, 50, 100}	$D\times$10,000
DE with Two Subpopulations [31]	Subpop.	30	300,000
MPEDE [58]	Subpop.	{30, 50}	$D\times$10,000
HMJCDE [22]	Subpop.	{30, 50}	$D\times$10,000
EDEV [59]	Subpop.	{30, 50}	$D\times$10,000
surrogate-assisted DEs			
CADE [28]	–	30	{10,000, 20,000}
CRADE [30]	–	{30, 500}	10,000
GPEME [26]	–	{20, 30, 50}	1,000
ESAO [56]	–	{20, 30, 50, 100, 200}	1,000
SAHO [40]	–	{10, 20, 30, 50, 100}	{110, 220, 330, 1,000}
DSS-DE [32]	–	{30, 50, 100}	1,000
SADE-ATDSC [38]	–	{10, 30, 50, 100}	1,000
DE-ABC [62]	Indiv.	{2, 3, 4, 6}	100,000
S-JADE [6]	Indiv.	{20, 30, 50, 100, 200}	{1,000, 1,500, 2,000}
SBSM-DE [21]	Indiv.	{10, 25, 60, 72, 942}	12,000
DESSA [29]	Indiv.	30	3,000
SMA-EPSDE [33]	Indiv.	{10, 30}	$D\times$10,000
ESMDE [34]	Indiv.	{10, 30}	$D\times$10,000
Sa-DE-DPS [11]	Indiv.	{10, 20, 30}	$D\times$ 50
SAPDE-ANN, SAPDE-RSM [1]	Indiv.	{10, 30}	$D\times$10,000
EBADE (Proposed Algorithm)	Subpop.	{10, 20, 30}	6,000

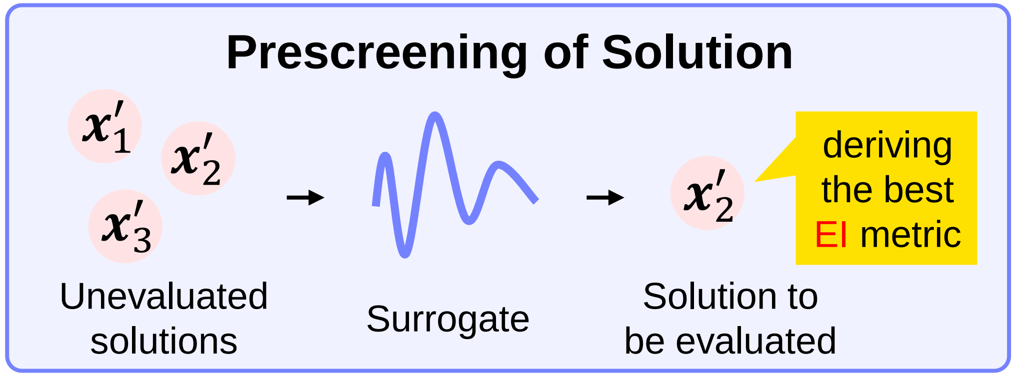
Columns "D" and "$FE_{\max}$" list the problem dimension and the maximum number of fitness evaluations adopted in the experiments, respectively.

Proposed Algorithm: EBADE
– Emulation-based Adaptive DE

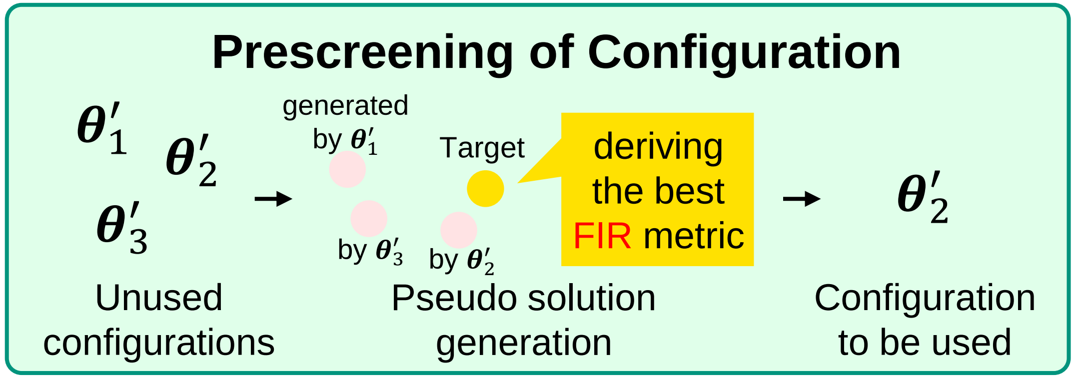
Concept

- **Emulating the efficient sampling method of SAEAs**
  - Prior Validation: prescreening “expected-to-improve” candidate

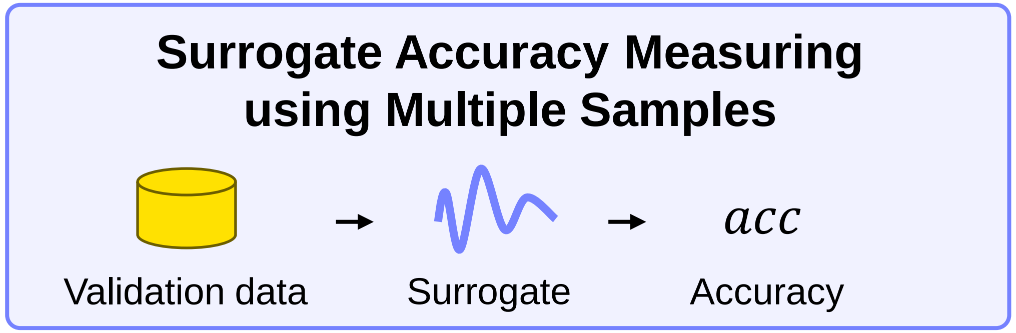
SAEA

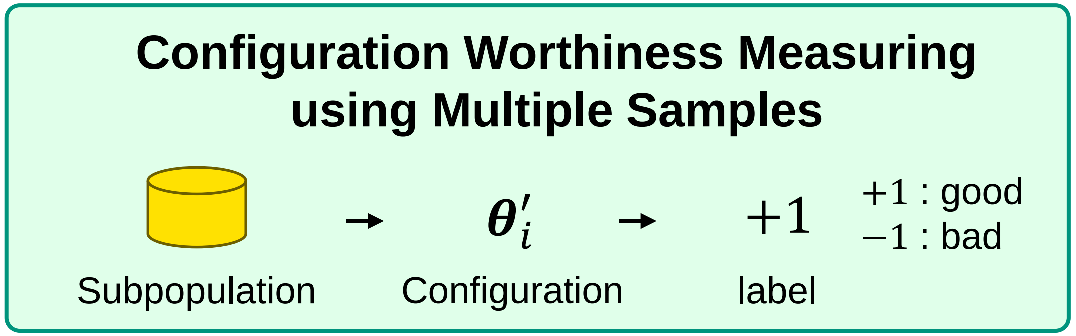


EBADE



  - Subpopulation-based Adaptation: validating with respect to multiple samples





Preliminary

- **Parameter configuration candidates to be adapted**
  - Numerical parameters
    - **Scaling Factor**: $\theta_F \in [0, 1]$
    - **Crossover Rate**: $\theta_{CR} \in [0, 1]$
  - Categorical parameters
    - **Mutation Strategy**: Right figure $\theta_v \in \{1, 2, 3, 4\}$
      - Four strategies are selected to accelerate exploitation.
    - **Crossover Strategy**: *binomial* and *exponential* (see p.6) $\theta_u \in \{1, 2\}$

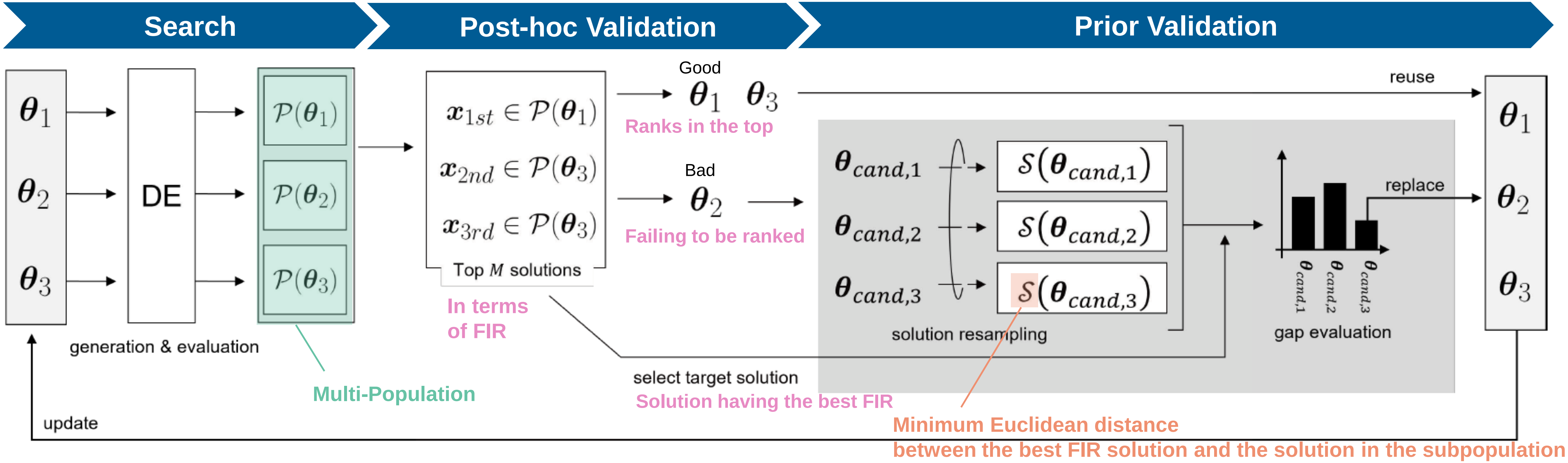
Each definition is in p.6

[Cai+ 13]

- **Initialization**
  - **Population**: Generate M subpopulations $\{\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_M\}$
    - Each subpopulation is composed of N solutions randomly generated in the search space.
  - **Configuration**: Generate M configuration vectors $\{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_M\}$ $\boldsymbol{\theta} = [\theta_F, \theta_{CR}, \theta_v, \theta_u]$
    - $\theta_F = 0.5, \theta_{CR} = 0.9$, θ_v and θ_u are randomly generated from their definitions.

Overall Framework

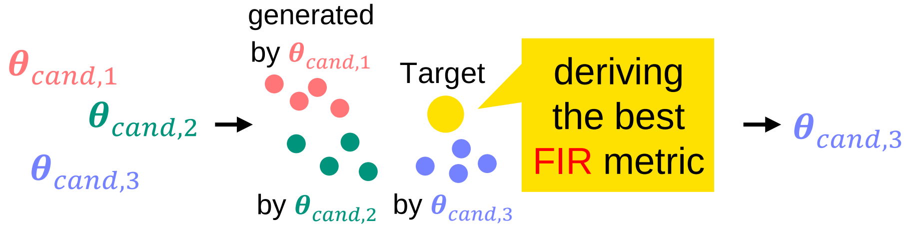
- **EBADE**
  - An example with $M = 3$ and $K = 3$, where K is the number of candidate $\boldsymbol{\theta}$s.



FIR: Fitness Improvement Ratio

$$\delta_f(\boldsymbol{x}_g) = 1 - \frac{f(\boldsymbol{x}_g)}{f(\boldsymbol{x}_{g-1}) + \delta_C}$$

- FIR is used to find the "expected-to-improve" solutions.
- $\delta_C \geq 0$ is a constant value to avoid division by 0.



Experiment

Experiment: Settings

Real-Parameter Single Objective Optimization Problem

- CEC 2013 benchmark suite (28 Problems, $D = \{10, 20, 30\}, FE_{\max} = \{2{,}000, 4{,}000, \mathbf{6{,}000}, 8{,}000, 10{,}000\}$)

Compared Algorithms and Their Configurations

Adaptive DEs

- **SHADE** [Tanabe+ 13] : $N = 100, M_{F,h,init} = M_{CR,h,init} = 0.5, F_{std} = CR_{std} = 0.1, H = 100, |Archive| = 100, p_{\min} = \frac{2}{N}, p_{\max} = 0.2$
- **jSO** [Brest+ 17] : $N_{init} = 25 \log D^{\frac{3}{2}}, N_{\min} = 4, M_{F,h,init} = 0.3, M_{CR,h,init} = 0.8, M_{F,H} = M_{CR,H} = 0.9, F_{std} = CR_{std} = 0.1, gen_{F,sep} = 0.6, F_{sep} = 0.7, F_{fix} = 0.7, gen_{CR,sep} = [0.25,0.5], CR_{maxcand} = [0.7,0.6], gen_{mut,sep} = [0.2,0.4], weight_{mut} = [0.7,0.8,1.2], H = 5, |Archive| = N, p_{\min} = 0.125, p_{\max} = 0.25$
- **CSDE** [Sun+ 20] : $N = 100, F_{init} = 0.5, CR_{init} = 0.5, FP = 200, \mu = 0.5, \sigma = 0.1$
- **EDEV** [Wu+ 18] : $\lambda_1 = \lambda_2 = \lambda_3 = 0.1, \lambda_4 = 0.7, ng = 20$

Surrogate-assisted DEs

- **GPEME** [Liu+ 14] : $N = 100, F = 0.8, CR = 0.8, \tau = 100, \lambda = 50, l = 4, \omega = 2, regr_{Kriging} = constant, corr_{Kriging} = gauss, \theta \in [10^{-5}, 10^2], \theta_{init} = 10^{-2}. D_{sammon,sep} = 50$
- **S-JADE** [Cai+ 19] : $N = 100, F_{out} = 0.5, CR_{out} = 0.75, p_{pbest\_out} = 0.05, F_{in} = 0.5, CR_{out} = 0.5, p_{pbest\_in} = 0.1, std_F = 0.1, std_{CR} = 0.1, L = 10, \epsilon = 0.01, c = 0.1, evals_{in} = 2{,}000, kernel_{RBF} = cubic, r = rand(0, 1.25)$
- **SAHO** [Pan+ 21] : $N = 100, F = 0.5, CR = 0.9, K = 30, neighbour = 5D (D < 50)\ or\ D (D \geq 50), kernel_{RBF} = cubic$
- **ESMDE** [Mallipeddi+ 15] : $N = 100, F \in [0.5,1.0], CR \in [0,1], mut \in [r/1\ c - \mathrm{t} - \mathrm{r}/1], xov \in [bin, exp], c = 10, regr_{Kriging} = constant, corr_{Kriging} = gauss, \theta \in [10^{-5}, 10^2], \theta_{init} = 10^{-2}$

Proposed Algorithms

- **EBADE** : $N = 4, M = 25, K = 6, p = 0.5$

Evaluation Metrics

- Average over 21 trials of the best fitness value
- Wilcoxon signed-rank test
- Average rank

Experiment: Results (Fitness at 6,000FEs)

- **EBADE outperforms adaptive DEs and is highly competitive with SAEAs**
  - ➢ EBADE is superior with statistical significance (the number of “-” is 9 to 27)
  - ➢ The best average rank is obtained by EBADE in all dimensions.

Best value

Worst value

+/−/∼ in Wilcoxon test
+ ：ours underperforms
− ：ours outperforms
∼ ：cannot find significance

$D = 10$

	Adaptive DEs					SAEAs			
	EBADE	SHADE	jSO	CSDE	EDEV	GPEME	S-JADE	SAHO	ESMDE
F1	4.69E-05	7.27E-01 –	1.13E-03 –	5.22E+00 –	4.27E+02 –	0.00E+00 +	3.03E-13 +	1.66E-28 +	1.81E-02 –
F2	2.18E+05	3.87E+06 –	2.24E+03 +	9.48E+06 –	9.20E+06 –	3.14E+06 –	1.14E+05 +	4.09E+04 +	7.40E+06 –
F3	6.46E+03	3.38E+06 –	7.57E+03 ∼	4.28E+07 –	8.37E+08 –	7.93E+07 –	5.07E+09 –	2.21E+10 –	7.76E+06 –
F4	6.90E+02	1.82E+04 –	1.09E+01 +	3.15E+04 –	2.52E+04 –	2.27E+04 –	9.12E+03 –	1.21E+04 –	2.48E+04 –
F5	8.49E-03	1.43E+00 –	1.15E-02 ∼	5.22E+00 –	1.82E+02 –	0.00E+00 +	1.73E+00 ∼	9.17E-05 +	2.98E-01 –
F6	1.50E+01	1.02E+01 ∼	8.65E+00 ∼	1.35E+01 ∼	5.25E+01 –	5.81E+00 ∼	8.41E+00 ∼	7.46E+00 ∼	9.90E+00 ∼
F7	1.26E+01	2.08E+01 –	3.83E+00 +	4.08E+01 –	6.04E+01 –	3.54E+01 –	7.41E+01 –	2.45E+02 –	3.67E+01 –
F8	2.06E+01	2.06E+01 ∼	2.05E+01 ∼	2.06E+01 ∼	2.06E+01 ∼	2.06E+01 ∼	2.06E+01 ∼	2.06E+01 ∼	2.06E+01 ∼
F9	3.93E+00	8.62E+00 –	7.74E+00 –	9.66E+00 –	9.53E+00 –	4.67E+00 –	4.50E+00 ∼	5.40E+00 –	8.99E+00 –
F10	6.43E-01	5.73E+00 –	5.24E-01 ∼	2.70E+01 –	1.32E+02 –	1.67E-01 +	8.62E-02 +	4.59E-01 +	3.46E+00 –
F11	6.16E+00	2.61E+01 –	2.25E+01 –	3.09E+01 –	4.92E+01 –	1.49E+01 –	1.05E+01 –	2.99E+01 –	2.07E+01 –
F12	1.83E+01	4.30E+01 –	3.01E+01 –	5.10E+01 –	6.74E+01 –	2.34E+01 ∼	1.88E+01 ∼	2.71E+01 –	4.73E+01 –
F13	2.38E+01	4.47E+01 –	3.20E+01 –	4.86E+01 –	6.61E+01 –	3.42E+01 –	2.38E+01 ∼	4.80E+01 –	4.63E+01 –
F14	1.83E+02	1.28E+03 –	1.18E+03 –	1.39E+03 –	1.32E+03 –	7.76E+02 –	3.47E+02 –	1.14E+03 –	1.18E+03 –
F15	1.08E+03	1.77E+03 –	1.64E+03 –	1.91E+03 –	1.86E+03 –	1.21E+03 ∼	1.15E+03 ∼	1.31E+03 ∼	1.85E+03 –
F16	1.53E+00	1.95E+00 –	1.78E+00 –	1.78E+00 ∼	1.83E+00 –	1.80E+00 –	1.58E+00 ∼	1.20E+00 +	1.86E+00 –
F17	2.00E+01	3.93E+01 –	3.97E+01 –	4.05E+01 –	7.69E+01 –	2.31E+01 ∼	2.27E+01 ∼	2.46E+01 –	3.33E+01 –
F18	3.80E+01	5.57E+01 –	4.96E+01 –	5.87E+01 –	8.92E+01 –	3.57E+01 ∼	3.74E+01 ∼	2.66E+01 +	5.69E+01 –
F19	1.74E+00	3.85E+00 –	2.52E+00 –	3.94E+00 –	1.01E+01 –	4.54E+00 –	8.43E+00 –	1.84E+00 ∼	3.75E+00 –
F20	3.61E+00	3.84E+00 –	3.45E+00 ∼	4.05E+00 –	4.10E+00 –	3.63E+00 ∼	3.75E+00 ∼	4.04E+00 –	4.08E+00 –
F21	3.81E+02	4.00E+02 –	4.00E+02 –	4.00E+02 –	4.43E+02 –	3.91E+02 –	4.17E+02 –	4.31E+02 ∼	3.81E+02 ∼
F22	3.06E+02	1.43E+03 –	1.45E+03 –	1.30E+03 –	1.45E+03 –	8.79E+02 –	4.78E+02 –	1.50E+03 –	1.34E+03 –
F23	1.44E+03	1.97E+03 –	1.77E+03 –	2.09E+03 –	2.06E+03 –	1.30E+03 ∼	1.26E+03 +	1.79E+03 –	1.93E+03 –
F24	2.13E+02	2.18E+02 –	2.10E+02 +	2.22E+02 –	2.24E+02 –	2.14E+02 ∼	2.14E+02 ∼	2.17E+02 –	2.22E+02 –
F25	2.13E+02	2.20E+02 –	2.11E+02 ∼	2.24E+02 –	2.23E+02 –	2.15E+02 ∼	2.14E+02 ∼	2.19E+02 –	2.23E+02 –
F26	1.65E+02	1.78E+02 ∼	1.94E+02 ∼	2.01E+02 –	1.98E+02 –	1.85E+02 ∼	1.53E+02 ∼	1.97E+02 –	1.93E+02 –
F27	4.23E+02	4.87E+02 –	4.44E+02 ∼	5.66E+02 –	5.81E+02 –	5.03E+02 –	4.35E+02 ∼	5.16E+02 –	5.45E+02 –
F28	4.12E+02	3.55E+02 ∼	3.01E+02 +	4.13E+02 ∼	6.40E+02 –	2.90E+02 +	7.21E+02 –	1.08E+03 –	3.27E+02 ∼
+/−/∼		0/24/4	5/14/9	0/24/4	0/27/1	4/13/11	4/9/15	6/17/5	0/24/4
Ave. rank	2.45	5.59	3.62	7.41	8.23	3.46	3.59	4.88	5.77

$D = 30$

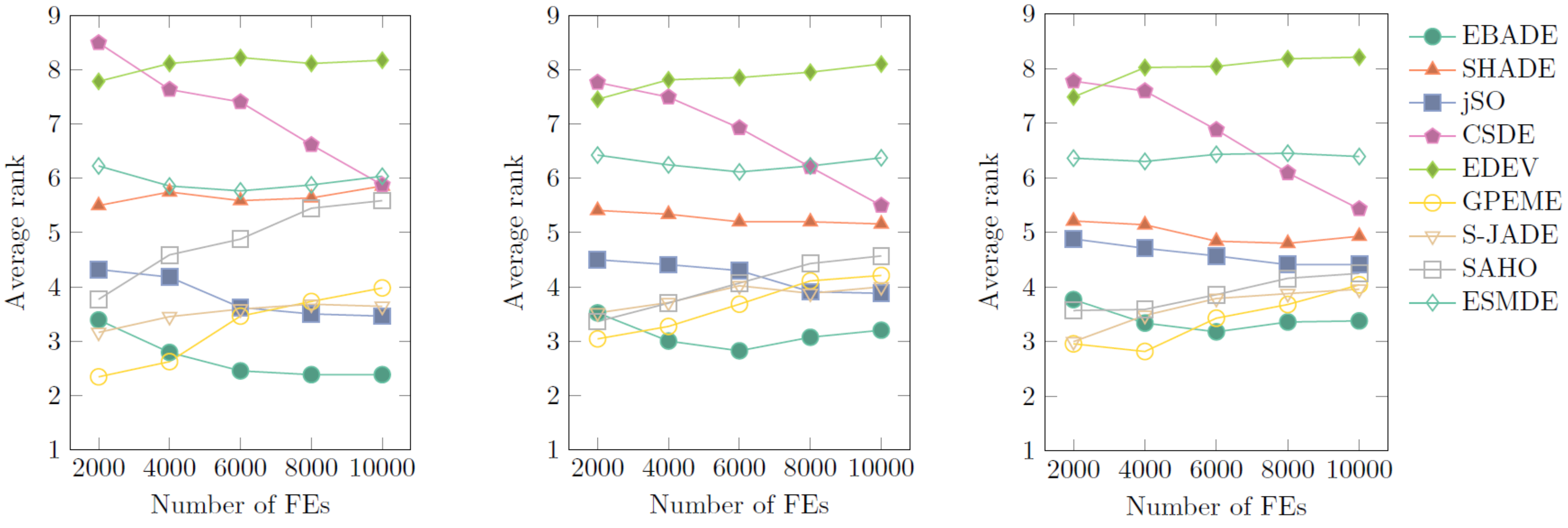
Adaptive DEs					SAEAs			
EBADE	SHADE	jSO	CSDE	EDEV	GPEME	S-JADE	SAHO	ESMDE
1.59E+02	2.39E+02 –	3.27E+02 –	1.06E+03 –	9.01E+03 –	5.44E+02 ∼	3.08E-07 +	7.24E-27 +	6.34E+02 –
2.96E+07	9.54E+07 –	1.63E+07 +	2.31E+08 –	3.00E+08 –	2.36E+07 ∼	5.74E+06 +	4.77E+05 +	1.87E+08 –
1.07E+10	1.00E+10 ∼	1.88E+10 –	5.76E+10 –	1.13E+11 –	6.43E+10 –	1.95E+11 –	1.03E+15 –	1.01E+11 –
3.71E+04	9.63E+04 –	3.85E+04 ∼	1.32E+05 –	1.26E+05 –	1.29E+05 –	6.56E+04 –	6.95E+04 –	1.17E+05 –
1.95E+02	2.10E+02 ∼	5.69E+02 –	7.27E+02 –	8.92E+03 –	9.42E+02 ∼	2.07E+02 ∼	1.87E-04 +	6.50E+02 –
1.37E+02	1.28E+02 ∼	1.34E+02 ∼	2.23E+02 –	1.11E+03 –	3.88E+01 +	4.37E+01 +	1.43E+01 +	2.23E+02 –
1.56E+02	1.30E+02 +	1.29E+02 +	1.94E+02 –	2.78E+02 –	2.29E+02 –	2.35E+02 –	7.76E+03 –	2.37E+02 –
2.11E+01	2.11E+01 ∼	2.11E+01 ∼	2.11E+01 ∼	2.11E+01 ∼	2.11E+01 ∼	2.11E+01 ∼	2.11E+01 ∼	2.11E+01 ∼
3.11E+01	4.00E+01 –	4.09E+01 –	4.27E+01 –	4.19E+01 –	2.28E+01 +	2.56E+01 +	3.24E+01 ∼	3.98E+01 –
1.52E+02	1.94E+02 –	1.04E+02 +	8.26E+02 –	2.57E+03 –	1.77E+02 ∼	1.31E+00 +	5.18E-02 +	8.63E+02 –
1.18E+02	2.04E+02 –	2.25E+02 –	2.18E+02 –	3.99E+02 –	9.87E+01 ∼	1.50E+02 –	2.68E+02 –	2.17E+02 –
1.97E+02	2.46E+02 –	2.27E+02 –	2.73E+02 –	4.50E+02 –	1.23E+02 +	1.85E+02 ∼	2.41E+02 ∼	2.93E+02 –
2.65E+02	2.47E+02 ∼	2.39E+02 +	2.77E+02 ∼	4.27E+02 –	2.18E+02 +	2.34E+02 +	2.54E+02 ∼	2.93E+02 –
3.75E+03	6.81E+03 –	7.73E+03 –	6.60E+03 –	6.56E+03 –	4.04E+03 ∼	4.34E+03 ∼	4.48E+03 –	6.13E+03 –
7.04E+03	8.14E+03 –	8.27E+03 –	8.36E+03 –	8.43E+03 –	7.79E+03 –	6.83E+03 ∼	5.12E+03 +	8.14E+03 –
3.24E+00	3.52E+00 ∼	3.65E+00 –	3.63E+00 –	3.74E+00 –	3.48E+00 ∼	3.71E+00 –	3.13E+00 ∼	3.68E+00 –
2.00E+02	2.53E+02 –	2.70E+02 –	2.98E+02 –	5.84E+02 –	1.31E+02 +	1.76E+02 ∼	1.40E+02 +	2.89E+02 –
2.97E+02	2.95E+02 ∼	2.77E+02 +	3.32E+02 –	6.01E+02 –	2.76E+02 +	2.18E+02 +	1.40E+02 +	3.37E+02 –
3.24E+01	2.63E+01 ∼	2.37E+01 +	1.06E+02 –	4.05E+04 –	1.99E+01 +	1.27E+02 –	1.44E+01 +	4.67E+02 –
1.44E+01	1.49E+01 –	1.43E+01 ∼	1.48E+01 –	1.48E+01 –	1.38E+01 +	1.46E+01 ∼	1.48E+01 –	1.50E+01 –
6.47E+02	7.05E+02 ∼	8.23E+02 –	1.34E+03 –	2.22E+03 –	2.20E+03 –	2.38E+03 –	3.59E+03 –	1.38E+03 –
3.92E+03	7.37E+03 –	8.22E+03 –	7.52E+03 –	7.70E+03 –	4.55E+03 ∼	5.46E+03 –	5.27E+03 –	6.87E+03 –
7.58E+03	8.82E+03 –	8.76E+03 –	8.87E+03 –	8.71E+03 –	7.51E+03 ∼	7.33E+03 ∼	5.68E+03 +	8.72E+03 –
2.79E+02	2.92E+02 –	2.86E+02 –	3.04E+02 –	3.12E+02 –	2.62E+02 +	2.85E+02 –	2.82E+02 ∼	3.03E+02 –
2.97E+02	3.13E+02 –	3.11E+02 –	3.19E+02 –	3.23E+02 –	2.79E+02 +	3.04E+02 –	3.00E+02 ∼	3.16E+02 –
2.74E+02	2.72E+02 ∼	2.17E+02 ∼	3.50E+02 –	3.07E+02 ∼	3.22E+02 ∼	2.35E+02 ∼	3.05E+02 ∼	2.52E+02 ∼
1.04E+03	1.26E+03 –	1.23E+03 –	1.36E+03 –	1.37E+03 –	8.88E+02 +	9.96E+02 ∼	1.07E+03 ∼	1.32E+03 –
1.34E+03	1.14E+03 ∼	1.46E+03 ∼	1.87E+03 –	2.93E+03 –	2.13E+03 –	3.07E+03 –	6.38E+03 –	2.08E+03 –
	1/16/11	6/16/6	0/26/2	0/26/2	11/6/11	7/11/10	10/9/9	0/26/2
3.18	4.84	4.57	6.88	8.04	3.43	3.79	3.86	6.43

Experiment: Results (Summary)

$+/-/\sim$ **in Wilcoxon test**
$+$: ours underperforms
$-$: ours outperforms
$\sim$: cannot find significance

- **Usefulness of EBADE and Limitation of SAEA in M-EOPs**
  - EBADE keeps deriving the best performance after 6,000 FEs, i.e., M-EOPs
  - The ranks of SAEAs decreases as the increase of the number of FEs.
  - Some adaptive DEs becomes effective as the increase of the number of FEs.
    (jSO and CSDE)

Average rank



a) $D = 10$ b) $D = 20$ c) $D = 30$

Wilcoxon signed-rank test

a) $D = 10$

FEs	vs SHADE	vs jSO	vs CSDE	vs EDEV	vs GPEME	vs S-JADE	vs SAHO	vs ESMDE
2,000	0/24/ 4	7/ 9/12	0/26/ 2	0/26/ 2	17/ 6/ 5	14/ 4/10	10/ 5/13	0/25/ 3
4,000	0/23/ 5	5/15/ 8	0/26/ 2	0/26/ 2	7/ 9/12	5/ 8/15	7/13/ 8	0/24/ 4
6,000	0/24/ 4	5/14/ 9	0/24/ 4	0/27/ 1	4/13/11	4/ 9/15	6/17/ 5	0/24/ 4
8,000	1/24/ 3	6/13/ 9	1/24/ 3	0/26/ 2	4/16/ 8	3/ 9/16	2/18/ 8	0/24/ 4
10,000	0/24/ 4	6/13/ 9	0/25/ 3	0/26/ 2	4/18/ 6	3/ 9/16	3/19/ 6	0/24/ 4

b) $D = 20$

FEs	vs SHADE	vs jSO	vs CSDE	vs EDEV	vs GPEME	vs S-JADE	vs SAHO	vs ESMDE
2,000	0/22/ 6	0/ 8/20	0/25/ 3	0/24/ 4	18/ 6/ 4	16/ 6/ 6	18/ 6/ 4	0/24/ 4
4,000	0/24/ 4	4/13/11	0/25/ 3	0/25/ 3	12/ 8/ 8	8/10/10	11/12/ 5	0/25/ 3
6,000	0/21/ 7	5/14/ 9	0/25/ 3	0/25/ 3	6/12/10	7/11/10	10/14/ 4	0/26/ 2
8,000	1/21/ 6	7/14/ 7	0/23/ 5	0/25/ 3	4/11/13	6/11/11	8/16/ 4	0/26/ 2
10,000	2/19/ 7	8/14/ 6	1/21/ 6	0/25/ 3	3/10/15	6/10/12	7/16/ 5	0/25/ 3

c) $D = 30$

FEs	vs SHADE	vs jSO	vs CSDE	vs EDEV	vs GPEME	vs S-JADE	vs SAHO	vs ESMDE
2,000	0/22/ 6	0/13/15	0/26/ 2	1/25/ 2	18/ 6/ 4	17/ 5/ 6	18/ 6/ 4	0/25/ 3
4,000	1/18/ 9	3/14/11	0/26/ 2	0/26/ 2	16/ 6/ 6	13/ 9/ 6	12/ 8/ 8	0/25/ 3
6,000	1/16/11	6/16/ 6	0/26/ 2	0/26/ 2	11/ 6/11	7/11/10	10/ 9/ 9	0/26/ 2
8,000	5/13/10	5/14/ 9	1/23/ 4	0/25/ 3	10/ 9/ 9	8/12/ 8	10/13/ 5	0/26/ 2
10,000	7/14/ 7	7/14/ 7	1/22/ 5	0/25/ 3	7/10/11	6/13/ 9	9/13/ 6	0/24/ 4

Experiment: Computational Time

- **Average runtime [sec] required to complete one trial (6,000 FEs)**
  - The runtime of EBADE is slightly longer than those of adaptive DEs.
    - However, this is not cared in M-EOPs.
  - The runtime of EBADE is much faster than those of SAEAs.
    - These long runtime of SAEAs are not accepted in M-EOPs.

		Adaptive DEs				SAEAs			
D	EBADE	SHADE	jSO	CSDE	EDEV	GPEME	S-JADE	SAHO	ESMDE
10	1.42E+01	1.18E+01	6.95E+00	1.35E+01	1.58E+01	3.30E+03	1.04E+05	4.13E+04	2.98E+03
20	1.45E+01	1.18E+01	6.98E+00	1.37E+01	1.80E+01	1.54E+04	1.11E+05	7.71E+04	1.53E+04
30	1.55E+01	1.21E+01	7.23E+00	1.44E+01	1.68E+01	3.34E+04	7.72E+04	8.65E+04	3.67E+04

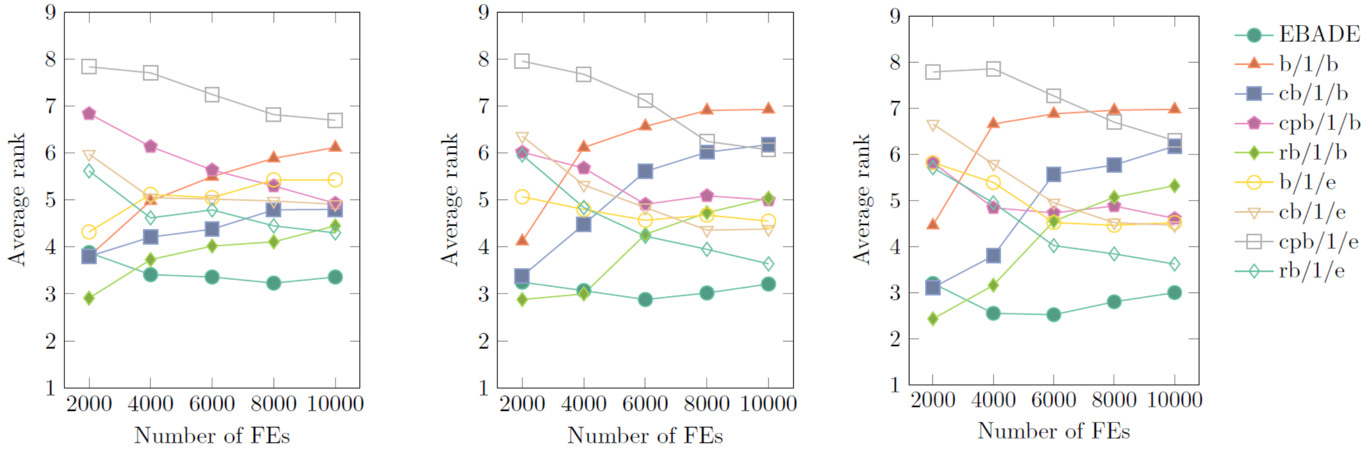
Discussion

Discussion 1/3

- **Impact of parameter adaptation in M-EOPs**
  - EBADE is compared with DEs with fixed parameter configuration.
    - Eight variants ($\theta_F = 0.5, \theta_{CR} = 0.9$)

		mutation			
		best/1	***current-to-best/1***	***current-to-pbest/1***	***rand-to-best/1***
crossover	***binomial***	b/1/b	cb/1/b	cpb/1/b	rb/1/b
	exponential	b/1/e	cb/1/e	cpb/1/e	rb/1/e

    - Result (average rank)
      - **EBADE is in top rank with 6,000 FEs and more.**
      - cb/1/b and rb/1/b are in high rank under 4,000 FEs. However, their rank degrades with a greater number of FEs.



a) D = 10 b) D = 20 c) D = 30

Conclusion The effectiveness of parameter adaptation of EBADE is confirmed.

Discussion 2/3

- **Parameter analysis for K (# of candidate $\boldsymbol{\theta}$s) and M (# of subpopulations)**
  - Ablation studies of the prior validation and multi-population can also be conducted.

Can be turned off by setting $K = 1$

Ditto by setting $M = 100$

+/−/∼ in Wilcoxon test
+ : default setting underperforms
− : default setting outperforms
∼ : cannot find significance

- Result
  - The performance of EBADE is sensitive to K and M.
  - The default setting outperforms or is competitive with the others.
  - EBADE with $K = 1$ or $M = 100$ clearly underperform the others.

a) $D = 10$

FEs	vs $K = 1$ (w/o PV)	vs $K = 2$	vs $K = 4$	vs $K = 8$	vs $K = 10$
2,000	0/16/12	0/13/15	0/ 2/26	1/ 0/27	3/ 0/25
4,000	0/17/11	0/12/16	0/ 7/21	2/ 1/25	5/ 1/22
6,000	0/16/12	0/13/15	1/ 5/22	2/ 2/24	3/ 3/22
8,000	1/15/12	1/11/16	2/ 6/20	4/ 2/22	3/ 3/22
10,000	2/10/16	2/ 9/17	2/ 5/21	3/ 2/23	3/ 4/21

b) $D = 20$

FEs	vs $K = 1$ (w/o PV)	vs $K = 2$	vs $K = 4$	vs $K = 8$	vs $K = 10$
2,000	0/16/12	0/15/13	0/ 1/27	2/ 0/26	3/ 0/25
4,000	0/18/10	0/13/15	0/ 3/25	3/ 0/25	4/ 0/24
6,000	0/13/15	1/13/14	0/ 1/27	3/ 1/24	3/ 2/23
8,000	1/11/16	1/ 8/19	1/ 1/26	2/ 2/24	1/ 2/25
10,000	1/ 8/19	1/ 7/20	2/ 1/25	2/ 2/24	1/ 5/22

c) $D = 30$

FEs	vs $K = 1$ (w/o PV)	vs $K = 2$	vs $K = 4$	vs $K = 8$	vs $K = 10$
2,000	1/17/10	0/10/18	1/ 3/24	2/ 0/26	5/ 0/23
4,000	1/15/12	0/ 9/19	0/ 0/28	2/ 1/25	3/ 0/25
6,000	0/14/14	0/11/17	0/ 0/28	1/ 2/25	1/ 1/26
8,000	0/ 9/19	0/ 5/23	0/ 0/28	1/ 2/25	1/ 2/25
10,000	3/ 7/18	1/ 5/22	0/ 0/28	0/ 2/26	2/ 3/23

a) $D = 10$

FEs	vs $M = 2$	vs $M = 4$	vs $M = 5$	vs $M = 10$	vs $M = 20$	vs $M = 50$	vs $M = 100$ (Indiv.)
2,000	0/ 7/21	0/ 6/22	0/ 4/24	2/ 1/25	0/ 1/27	1/ 1/26	0/ 0/28
4,000	1/12/15	1/ 8/19	0/ 5/23	1/ 1/26	0/ 1/27	0/ 3/25	1/ 3/24
6,000	0/15/13	0/10/18	0/ 7/21	0/ 1/27	0/ 1/27	0/ 3/25	1/ 4/23
8,000	0/12/16	0/ 7/21	0/ 7/21	0/ 0/28	0/ 0/28	1/ 4/23	0/ 6/22
10,000	2/11/15	0/ 8/20	0/ 6/22	0/ 0/28	0/ 1/27	0/ 5/23	0/ 6/22

b) $D = 20$

FEs	vs $M = 2$	vs $M = 4$	vs $M = 5$	vs $M = 10$	vs $M = 20$	vs $M = 50$	vs $M = 100$ (Indiv.)
2,000	0/10/18	0/ 1/27	0/ 4/24	0/ 0/28	0/ 0/28	1/ 4/23	0/ 1/27
4,000	0/13/15	0/ 2/26	1/ 4/23	0/ 1/27	1/ 1/26	0/ 1/27	1/ 1/26
6,000	0/ 9/19	0/ 2/26	1/ 3/24	2/ 1/25	2/ 1/25	0/ 2/26	1/ 3/24
8,000	0/ 8/20	1/ 5/22	1/ 3/24	1/ 0/27	2/ 0/26	0/ 2/26	0/ 5/23
10,000	1/ 6/21	1/ 7/20	1/ 4/23	2/ 1/25	1/ 1/26	0/ 2/26	0/ 4/24

c) $D = 30$

FEs	vs $M = 2$	vs $M = 4$	vs $M = 5$	vs $M = 10$	vs $M = 20$	vs $M = 50$	vs $M = 100$ (Indiv.)
2,000	0/ 8/20	0/ 5/23	0/ 3/25	3/ 1/24	1/ 1/26	0/ 1/27	2/ 2/24
4,000	0/ 9/19	0/ 2/26	0/ 2/26	0/ 1/27	1/ 1/26	0/ 1/27	1/ 1/26
6,000	0/10/18	0/ 4/24	0/ 4/24	0/ 1/27	0/ 1/27	0/ 3/25	1/ 3/24
8,000	1/12/15	1/ 4/23	1/ 4/23	0/ 1/27	0/ 1/27	0/ 2/26	1/ 6/21
10,000	1/10/17	1/ 4/23	2/ 4/22	0/ 1/27	0/ 0/28	0/ 2/26	1/ 5/22

Conclusion The prior validation and multi-population mechanisms are necessary.

- **Adaptation result**
  - ➢ The ratio of each candidate used
    - ■ Shown by problem function and the dimension.
    - ■ Result
      - All candidates for each configuration are selected avoiding strong bias.
    - ■ Analysis example
      - CR prefers higher values.
        - ✓ Solutions generated by exploitation mutation strategies should be actively utilized in EOPs.
      - Exploitation-oriented mutation strategy (*best/1*) is most frequently selected in EOPs.

Conclusion EBADE exhibited high performance by selecting more candidates appropriate for EOPs.

Conclusion

Conclusion

- **Emulation-based Adaptive DE for M-EOPs**
  - EBADE emulates sample-efficient approaches like SAEAs.
    - Prior validation mechanism prescreens candidate configurations without FEs.
    - Multi-population mechanism validates candidate configurations with respect to multiple samples.
  - **High performance, Fast, and Auto-tunable**
    - Outperforming adaptive DEs and highly competitive with SAEAs.
    - Much shorter runtime than those of SAEAs.
    - Automatic performance improvement and easy-to-use
- **Future Work**
  - Extension to multi-objective EOPs
  - Development of solution screening mechanism without using any ML technique.

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